Introduction

In this section, the lessons focus on creating sample spaces and using the Fundamental (Basic) Counting Principle. Students calculate experimental probability through games and then calculate the theoretical probability. These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. This is one method to help students connect the content from grade to grade and to accelerate.

For the lessons in this section, you will need the materials listed at right.

MATERIALS SUMMARY
Scientific calculators
Paper clips
Number cubes
Index cards
Round, colored plastic chips
Colored pencils, crayons, or markers

Standards of Learning

5.17 The student will
- a) solve problems involving the probability of a single event by using tree diagrams or by constructing a sample space representing all possible results;
- b) predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction; and
- c) create a problem statement involving probability and based on information from a given problem situation. Students will not be required to solve the created problem statement.

6.20 The student will
- a) make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram; and
- b) determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal, or percent, as appropriate for the given situation.

7.14 The student will investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.

7.15 The student will identify and describe the number of possible arrangements of several objects, using a tree diagram or the Fundamental (Basic) Counting Principle.

8.11 The student will analyze problem situations, including games of chance, board games, or grading scales, and make predictions, using knowledge of probability.

Table of Contents

The following lessons are included in this section. Click (or CTRL+click) on each to jump to that lesson.

✱ SOL 5.17a, b, c

Prerequisite SOL
4.19

Lesson Summary
Students solve single-event probability problems, using number cubes, and reach solutions through the use of the sample space. They represent in fraction and decimal forms predictions related to the probability of outcomes in these problems. (50–60 minutes)

Materials
Number cubes
Calculators
Copies of the attached worksheets

Vocabulary
sample space. A set of all possible outcomes of an activity or experiment.

prediction. A reasonable guess as to what might happen in a certain situation, based on data gathered from a sample space.

probability. The chance that a certain outcome will occur, written as a ratio of the number of favorable outcomes divided by the number of possible outcomes; may be displayed as a ratio (fraction), decimal, or percent.

Warm-up
Allow students about five minutes to complete the "Will It Happen?" worksheet. Then, hold a class discussion about the students' responses, having them share their answers and asking them to explain their reasoning. Some students may say that the world could vanish tomorrow or that a wizard could change all red colors to green. In that case, remind them that they are dealing with the real world, not the world of fantasy. Correct answers will vary, but focus on the students' reasoning. Guide the discussion so that all students agree that knowledge about an event can help a person make a prediction about the probability of that event occurring. Tell the class that in this lesson, they will focus on ways to display information so that the probability of an event's occurring can be predicted more accurately.

Lesson
1. Allow students to work with partners. Give each set of partners two number cubes and a copy of the "What's the Probability? Part 1" worksheet. Read the directions aloud to the class.
2. Give students sufficient time to complete the worksheet. As they finish, have one member of each set of partners transfer their results to a class record sheet on chart paper or the board.
3. Hold a class discussion to analyze the results displayed on the class record sheet. Lead students to discover that some numbers, e.g., 6, 7, and 8, appear more often than others. The numbers 5 and 9 might also appear a significant number of times, but students should recognize that the more times the experiment is conducted, the more frequently the numbers 6, 7, and 8 appear.
4. Have partners complete Part 2 of the worksheet. As students work, check for accuracy, and clear up mistakes. Have partners share and compare their findings.
5. Lead a class discussion of the results. The discussion should lead students to conclude that the Number-Cube Game, with the sums assigned as they are to each player, is unfair because Player B has more chances of winning. Include in the discussion the factor that makes a game fair: the chance of either player winning is ½, 0.50, or 50 percent.

Reflection
Ask students to describe in writing how they could change the rules of the game so that it would be fair. For example, they could change the rules so that Player A gets a point when the sum of the number cubes is an odd number, while Player B gets a point when the sum is even.

Name: ________________________

Will It Happen?

Listed below is a set of events. Some will definitely happen, some will never happen, and some may or may not happen. Place an *X* on the “Line of Certainty” shown below each event to indicate the chances of the event happening.

Example
You will have two birthdays this year.

X

Definitely Will Happen | Maybe Will Happen | Will Never Happen

1. The sun will set, and darkness will come over the Earth tomorrow.

Definitely Will Happen | Maybe Will Happen | Will Never Happen

2. Everything in the world that is colored red will turn to green today.

Definitely Will Happen | Maybe Will Happen | Will Never Happen

3. It will rain today.

Definitely Will Happen | Maybe Will Happen | Will Never Happen

4. When you flip a coin, it will land on heads.

Definitely Will Happen | Maybe Will Happen | Will Never Happen

5. You have a spinner that is divided into four equal parts with an “A” printed in one part, a “B” in another part, a “C” in another part, and a “D” in the last part. When you spin the spinner, it will land on “B.”

Definitely Will Happen | Maybe Will Happen | Will Never Happen

Name: ________________________

What's the Probability? Part 1

Know the Rules

Two friends created a number-cube game, using a pair of number cubes. They decided that Player A scores a point whenever the *sum* of a roll of the cubes is 2, 3, 4, 9, 10, 11, or 12. Player B scores a point whenever the *sum* is 5, 6, 7, or 8. For example, they roll the cubes, and one cube is 5 and the other is 4. Because the sum is 9, Player A scores a point. One game consists of rolling the cubes 25 times.

Predict the Results

Which player do you think will win the game?
Player _______

Play the Game to Verify Your Prediction

With your partner, take turns rolling a pair of number cubes 25 times. Using the rules stated above, record on the chart below the points earned. Use tally marks to record the results in each column.

Player A **(2, 3, 4, 9, 10, 11, 12)**	**Player B** **(5, 6, 7, 8)**

Player A got ______ points.

Player B got ______ points.

Express the number of points won by each player in 25 rolls as a ratio (fraction). Then, simplify the fraction, if possible, and finally express it as a decimal.

Player A: $\frac{\text{number of points}}{\text{25 rolls}} = \frac{\_\_}{25} = \frac{\_\_}{\_\_} =$ _______

Player B: $\frac{\text{number of points}}{\text{25 rolls}} = \frac{\_\_}{25} = \frac{\_\_}{\_\_} =$ _______

Name: <u>ANSWER KEY</u>

What's the Probability? Part 1

Know the Rules
Two friends created a number-cube game, using a pair of number cubes. They decided that Player A scores a point whenever the *sum* of a roll of the cubes is 2, 3, 4, 9, 10, 11, or 12. Player B scores a point whenever the *sum* is 5, 6, 7, or 8. For example, they roll the cubes, and one cube is 5 and the other is 4. Because the sum is 9, Player A scores a point. One game consists of rolling the cubes 25 times.

Predict the Results
Which player do you think will win the game?
Player <u>(Guesses will vary, but students will more likely guess Player A because there would seem to be more chances for Player A to win a point.)</u>

Play the Game to Verify Your Prediction
With your partner, take turns rolling a pair of number cubes 25 times. Using the rules stated above, record on the chart below the points earned. Use tally marks to record the results in each column.

<u>(Answers will vary, of course, but the results shown below are typical. Check to ensure that the fractions and decimals are written correctly.)</u>

Player A **(2, 3, 4, 9, 10, 11, 12)**	**Player B** **(5, 6, 7, 8)**

Player A got <u> 10 </u> points.

Player B got <u> 15 </u> points.

Express the number of points won by each player in 25 rolls as a ratio (fraction). Then, simplify the fraction, if possible, and finally express it as a decimal.

Player A: $\dfrac{\text{number of points}}{\text{25 rolls}} = \dfrac{10}{25} = \dfrac{2}{5} = \underline{\ \ 0.4\ \ }$

Player B: $\dfrac{\text{number of points}}{\text{25 rolls}} = \dfrac{15}{25} = \dfrac{3}{5} = \underline{\ \ 0.6\ \ }$

Name: ______________________________

What's the Probability? Part 2

Use the chart below to display *all possible outcomes* of rolling a pair of differently colored number cubes. One of the cubes is red and the other is yellow. The chart is partially completed. Fill in the empty boxes with the correct sums.

Then, shade in each box containing a sum that gives a point to Player A (2, 3, 4, 9, 10, 11, or 12). Do not shade the boxes containing sums that give points to Player B (5, 6, 7, or 8).

Possible Outcomes

	R1	R2	R3	R4	R5	R6
Y1	2					
Y2					7	
Y3						
Y4						
Y5			8			
Y6						12

The number of possible outcomes of rolling a pair of different colored cubes is _____ .

Player A can get a point with _____ of these _____ outcomes. Express this number as a ratio (fraction). Then, simplify the fraction, if possible, and finally express it as a decimal.

Player A: $\frac{\text{number of winning outcomes}}{\text{number of possible outcomes}} = \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_} = \_\_\_\_\_\_\_$

Player B can get a point with _____ of these _____ outcomes. Express this number as a ratio (fraction). Then, simplify the fraction, if possible, and finally express it as a decimal.

Player B: $\frac{\text{number of winning outcomes}}{\text{number of possible outcomes}} = \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_} = \_\_\_\_\_\_\_$

Compare the results of playing the game in Part 1 with the results of rolling the number cubes in Part 2. Share your results with those found by another set of partners. Be ready to participate in a class discussion of your findings.

Name: ANSWER KEY

What's the Probability? Part 2

Use the chart below to display *all possible outcomes* of rolling a pair of differently colored number cubes. One of the cubes is red and the other is yellow. The chart is partially completed. Fill in the empty boxes with the correct sums.

Then, shade in each box containing a sum that gives a point to Player A (2, 3, 4, 9, 10, 11, or 12). Do not shade the boxes containing sums that give points to Player B (5, 6, 7, or 8).

Possible Outcomes

	R1	R2	R3	R4	R5	R6
Y1	**2**	3	4	5	6	7
Y2	3	4	5	6	**7**	8
Y3	4	5	6	7	8	9
Y4	5	6	7	8	9	10
Y5	6	7	**8**	9	10	11
Y6	7	8	9	10	11	**12**

The number of possible outcomes of rolling a pair of different colored cubes is 36 .

Player A can get a point with 16 of these 36 outcomes. Express this number as a ratio (fraction). Then, simplify the fraction, if possible, and finally express it as a decimal.

Player A: $\dfrac{\text{number of winning outcomes}}{\text{number of possible outcomes}} = \dfrac{16}{36} = \dfrac{4}{9} = 0.444...$

Player B can get a point with _____ of these _____ outcomes. Express this number as a ratio (fraction). Then, simplify the fraction, if possible, and finally express it as a decimal.

Player B: $\dfrac{\text{number of winning outcomes}}{\text{number of possible outcomes}} = \dfrac{20}{36} = \dfrac{5}{9} = 0.555...$

Compare the results of playing the game in Part 1 with the results of rolling the number cubes in Part 2. Share your results with those found by another set of partners. Be ready to participate in a class discussion of your findings.

(Answers will vary, but all answers should indicate that Player B has more of a chance of winning, at least theoretically, because more of the possible outcomes will give points to Player B.)

✱ SOL 6.20

Prerequisite SOL

5.17a, b, c

Lesson Summary

Students create a sample space for a probability experiment, using a set of numbered cards. The sample space is represented as a list and as a tree diagram. (50–60 minutes)

Materials

Sets of red cards labeled 3X, 3Y, 3Z, 5X, 5Y
Sets of blue cards labeled 3X, 5X, 5Y
Calculators
Copies of the attached worksheets
Colored pencils, crayons, or markers

Vocabulary

probability. The chance that a certain outcome will occur, written as a ratio of the number of favorable outcomes divided by the number of possible outcomes; may be displayed as a ratio (fraction), decimal, or percent.

sample space. A set of all possible outcomes of an activity or experiment.

Warm-up

Hand out copies of the “Getting Ready for Picture Day” worksheet, and allow students about five minutes to complete it. Then, hold a class discussion about the students’ responses, allowing them to share their clothing choices with the class. Make a list on chart paper or the board of the clothes that were chosen. Use a list form in which tally marks can be used to record the number of times an article of clothing is selected by the class. Lead students into a discussion of the number of different choices that could have been made when they selected clothes. The set of all possible choices can be called a *sample space.* Tell the class that in this lesson, they will focus on ways that a sample space can be represented visually, and they will learn how to represent the probability of an event occurring, using a sample space.

Lesson

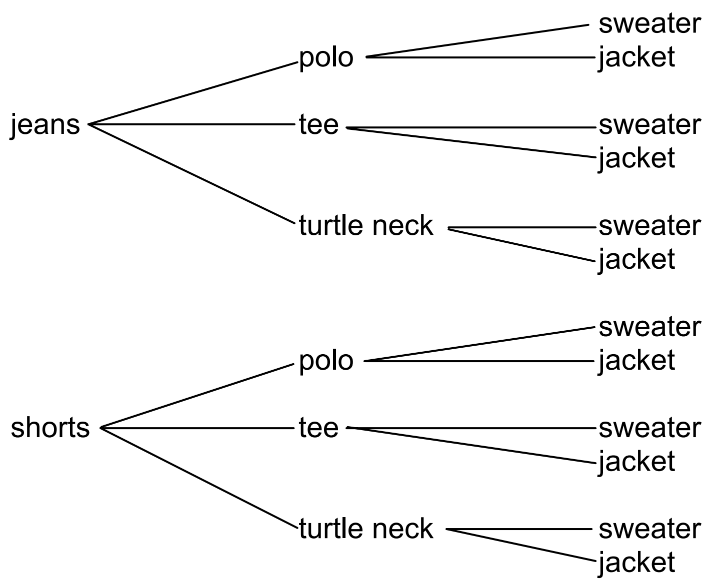
1. Give each pair of students a set of five red number cards and a set of three blue number cards. Point out that each card can be identified by its color, number, and letter. Have the partners display their cards on a desk or table by color and in alphanumeric order. Explain that the experiment will show all outcomes of pairing each of the red cards with each of the blue cards.
2. Distribute copies of the “Card-Choice Experiment” worksheet, and read the directions aloud. As partners complete the worksheet, move from one pair to another to monitor their work, noting any problems or discoveries made by the students, and taking time to clarify or to allow for sharing. Before students move to the second page of the activity, be sure that they have an understanding of the way the sample space was created and the way the probability of the two example events was determined. You may also need to review changing a fraction to a percent.
3. Explain that a *list* is a good way to organize items in a sample space but that sometimes a list may not be convenient or items may be left out of it accidentally. Tell students that another way to organize items in a sample space is to make a visual aid called a “tree diagram.” A common example of a tree diagram is the family tree structure used to display a person’s ancestors. The word *tree* is used to describe this type of display because the display resembles a tree with its trunk and branches that extend in an outward direction. The probability of an event occurring can also be determined through the use of a tree diagram.
4. Have students examine the tree diagram on page 2 of the handout. Tell them that it is used to organize the items in a sample space that shows the outcomes of flipping a coin twice. Next to the diagram is a list of the results for comparison. Ask students what would happen to the diagram if the coin were flipped a third time. Ensure understanding of tree-diagram construction through discussion.

5. Assign the completion of the “Card-Choice Experiment” worksheet. Move from one pair to another to clarify misunderstandings. After students are finished, display the results on a transparency or on the board. Discuss the results in relation to discovering the mathematical probability of an event.

Reflection

Ask students to return to their “Getting Ready for Picture Day” worksheet and to create a tree diagram of all of the possible ways that the clothes could have been chosen (see below). Have them draw the tree diagram next to the stick figure or on the back of the page, allowing them to work either with their partner or individually. Collect diagrams at the end of class, and evaluate them for understanding.

<u>Tree Diagram (Sample Space)</u>



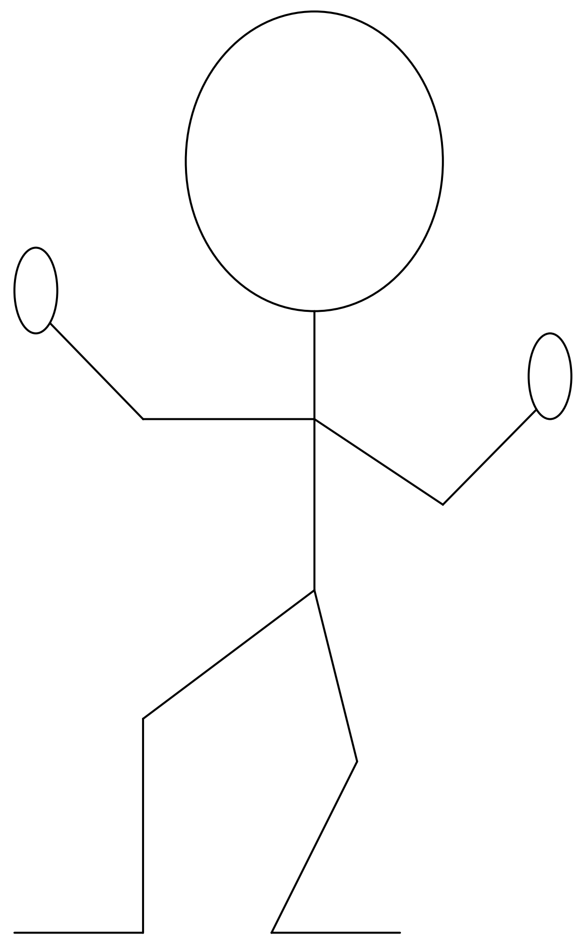
Name: ______________________

Getting Ready for Picture Day

Read the story below, and make decisions based on the choices that are given to you. Write your choices in the blanks provided. Be ready to explain why you made the choices that you did.

> *Today is the big day. It is picture day at school. You want to look your best, of course. You go to your closet and choose some clothes that might make you look just like you want to look. After a few minutes of taking clothes out of the closet and laying them on your bed, you are ready to make the decision. On the bed you have laid out a pair of faded jeans, a pair of khaki shorts, a white polo shirt, a blue tee shirt, a red turtle neck shirt, a gray sweater, and a dark blue jacket. You look at the jeans and the shorts and decide to wear the ______________________. Then you look at the polo shirt, the tee shirt, and the turtle neck shirt and decide to wear the ______________________. Finally, you look at the gray sweater and the blue jacket. You choose the ______________________.*

On the stick figure below, draw the clothes that you chose. Be sure to add the colors of choice to each article of clothing. Put shoes on the figure, and add a face and hair. Prepare to share your final choices with the class.



Name: ______________________

Card-Choice Experiment, page 1

1. Place the set of 5 red cards and the set of 3 blue cards on your desk in the following order:

Red	Red	Red	Red	Red	Blue	Blue	Blue
3X	3Y	3Z	5X	5Y	3X	5X	5Y

2. Make a list of all possible outcomes of pairing each red card with each blue card. The first set of outcomes is shown.

Red3X, Blue3X	Red3X, Blue5X	Red3X, Blue5Y
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________

3. Use the list above to find the probability of drawing a pair of 3s. Show the results as a ratio (fraction) and as a percent.

$$P(3, 3) = \frac{\text{number of favorable events}}{\text{number of possible events}} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

4. Use the list above to find the probability of drawing a pair of 5s. Show the results as a ratio (fraction) and as a percent.

$$P(5, 5) = \frac{\text{number of favorable events}}{\text{number of possible events}} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

Name: ________________________

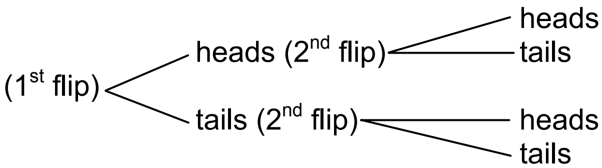
Card-Choice Experiment, page 2

Flipping the coin always results in either heads or tails. What are the possible outcomes of flipping a coin twice?

- If heads is the result of the first flip, then the possible outcomes of two flips are heads, heads or heads, tails.
- If tails is the result of the first flip, then the possible outcomes of two flips are tails, heads or tails, tails.

These results can be displayed as a tree diagram or as a list.

Tree Diagram (Sample Space)

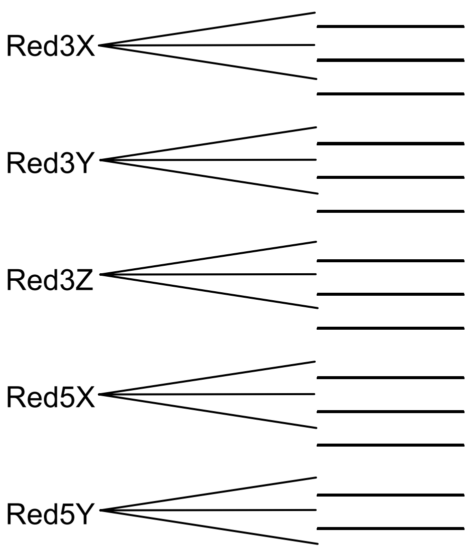


List of Possible Outcomes (Sample Space)

heads, heads
heads, tails

tails, heads
tails, tails

1. Create a tree diagram showing all possible outcomes of the Card-Choice Experiment.



2. Use your tree diagram to determine the following outcomes. Show the probability of each as a ratio (fraction) and as a percent.

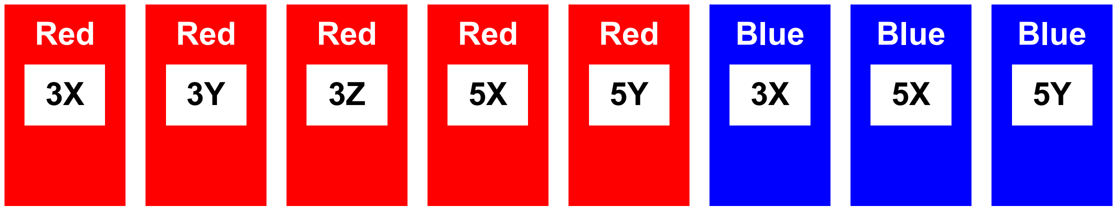
$P(3, 5) = \frac{\text{number of favorable events}}{\text{number of possible events}} =$ ______________________________

P(5, a number with an X) = ______________________________

Name: ANSWER KEY

Card-Choice Experiment, page 1

1. Place the set of 5 red cards and the set of 3 blue cards on your desk in the following order:



2. Make a list of all possible outcomes of pairing each red card with each blue card. The first set of outcomes is shown.

Red3X, Blue3X	Red3X, Blue5X	Red3X, Blue5Y
Red3Y, Blue3X	Red3Y, Blue5X	Red3Y, Blue5Y
Red3Z, Blue3X	Red3Z, Blue5X	Red3Z, Blue5Y
Red5X, Blue3X	Red5X, Blue5X	Red5X, Blue5Y
Red5Y, Blue3X	Red5Y, Blue5X	Red5Y, Blue5Y

3. Use the list above to find the probability of drawing a pair of 3s. Show the results as a ratio (fraction) and as a percent.

$$P(3, 3) = \frac{\text{number of favorable events}}{\text{number of possible events}} = \frac{3}{15} = 20\%$$

4. Use the list above to find the probability of drawing a pair of 5s. Show the results as a ratio (fraction) and as a percent.

$$P(5, 5) = \frac{\text{number of favorable events}}{\text{number of possible events}} = \frac{4}{15} = 26.7\%$$

Name: ANSWER KEY

Card-Choice Experiment, page 2

Flipping the coin always results in either heads or tails. What are the possible outcomes of flipping a coin twice?

- If heads is the result of the first flip, then the possible outcomes of two flips are heads, heads or heads, tails.
- If tails is the result of the first flip, then the possible outcomes of two flips are tails, heads or tails, tails.

These results can be displayed as a tree diagram or as a list.

Tree Diagram (Sample Space)			List of Possible Outcomes (Sample Space)
		heads	heads, heads
	heads (2nd flip)	tails	heads, tails
(1st flip)			
	tails (2nd flip)	heads	tails, heads
		tails	tails, tails

1. Create a tree diagram showing all possible outcomes of the Card-Choice Experiment.

Red3X	Blue3X
	Blue5X
	Blue5Y
Red3Y	Blue3X
	Blue5X
	Blue5Y
Red3Z	Blue3X
	Blue5X
	Blue5Y
Red5X	Blue3X
	Blue5X
	Blue5Y
Red5Y	Blue3X
	Blue5X
	Blue5Y

2. Use your tree diagram to determine the following outcomes. Show the probability of each as a ratio (fraction) and as a percent.

$$P(3, 5) = \frac{\text{number of favorable events}}{\text{number of possible events}} = \frac{8}{15} = 53.3\%$$

$$P(5, \text{a number with an X}) = \frac{6}{15} = \frac{2}{5} = 40\%$$

✱ SOL 7.14

Prerequisite SOL
5.17, 6.20

Lesson Summary
Students discover the difference between experimental probability and theoretical probability by comparing the theoretical probability of the results of an event with the actual results achieved when performing a series of trials of that event. Specifically, students compare the theoretical probability of the results of spinning a spinner a predetermined number of times with the actual results achieved when making the trials. Data from all students is used to make the sample. (50–60 minutes)

Materials
Calculators
Paper clips
Copies of the attached worksheets

Vocabulary
experimental probability. A statement of probability based on the results of performing a series of trials of a defined event.

theoretical probability. The predicted outcome of an event occurring, expressed as a ratio of the number of favorable outcomes divided by the number of possible outcomes; may be displayed as a ratio (fraction), decimal, or percent.

Warm-up
The purpose of the warm-up is to ensure that students understand how to use a calculator to change ratios (fractions) to decimals and decimals to percents. Allow students about five minutes to complete the "Parts of a Whole" worksheet. If necessary, discuss rounding to the nearest hundredth. Have students share their answers with the class, and have a class discussion of them. Be sure to correct any problems students may have in using a calculator for these purposes.

Lesson
1. Introduce the lesson by asking students how they know that something is fair. Give an example, such as, "How do you know that flipping a coin is a fair way to decide who gets to play a game first?" Lead the students to realize that some events have a mathematical outcome that is fair because every participant has *the same chance* of winning. Tell the class that this lesson will focus on how to find out if an event is fair by comparing its theoretical probability with its experimental probability. Review the definitions of *theoretical probability* and *experimental probability.*
2. Distribute copies of "The Spinner Game" worksheet, and do the first task together as a class. If you perceive that the students need another example, use the probability of rolling a 1 on one roll of a number cube numbered 1–6. Then, have students work with a partner to complete the second and third tasks on the sheet. Move among the groups and monitor the activity to ensure understanding of the tasks.
3. Discuss the results of tasks two and three after all groups have completed them. Focus on the similarities and differences between the theoretical probability and the experimental probability.
4. Read the directions for task four together as a class. Allow students time to exchange their data with each other. Monitor student work to ensure understanding. Then, encourage students to use complete sentences to do task five.
5. After each group finishes their writing, have them transfer their responses to a transparency or the board, and use these to prompt a closing class discussion. Ensure that everyone can express the difference between theoretical probability and experimental probability.

Reflection

Have students complete “Theoretical Probability and Experimental Probability Reflection” worksheet. Collect it from students before they leave class, and use it to evaluate student understanding.

Name: ________________________

Parts of a Whole

Use your scientific calculator to find the decimal equivalent of the ratios (fractions) listed below. Then change the decimal to a percent. Answers should be rounded to the nearest hundredth.

Example: $\frac{1}{2}$ = 0.50 = 50%

$\frac{1}{4}$ = ______ = ______% $\frac{1}{5}$ = ______ = ______%

$\frac{2}{7}$ = ______ = ______% $\frac{2}{3}$ = ______ = ______%

$\frac{1}{8}$ = ______ = ______% $\frac{3}{11}$ = ______ = ______%

$\frac{2}{9}$ = ______ = ______% $\frac{5}{6}$ = ______ = ______%

Name: ANSWER KEY

Parts of a Whole

Use your scientific calculator to find the decimal equivalent of the ratios (fractions) listed below. Then change the decimal to a percent. Answers should be rounded to the nearest hundredth.

Example: $\frac{1}{2}$ = 0.50 = 50%

$\frac{1}{4}$ = 0.25 = 25%

$\frac{1}{5}$ = 0.20 = 20%

$\frac{2}{7}$ = 0.29 = 29%

$\frac{2}{3}$ = 0.67 = 67%

$\frac{1}{8}$ = 0.13 = 13%

$\frac{3}{11}$ = 0.27 = 27%

$\frac{2}{9}$ = 0.22 = 22%

$\frac{5}{6}$ = 0.83 = 83%

Name: ________________________

The Spinner Game

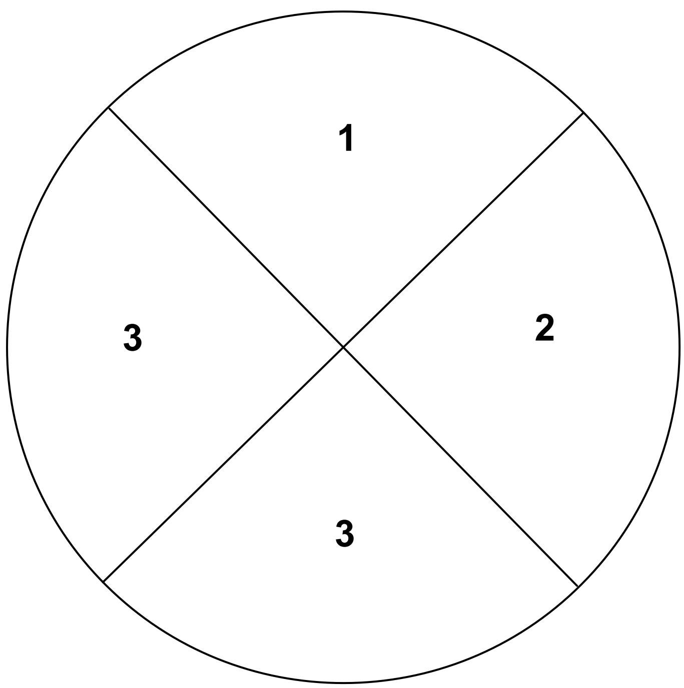
A spinner divided into four congruent parts is used in this game. (See the spinner below.) When the paperclip pointer lands on 3, the player earns a point.

1. What is the *theoretical probability* of spinning a 3? Use the formula below to calculate the theoretical probability of this event, and show the answer as a ratio, decimal (rounded to the nearest hundredth), and percent.

   Theoretical Probability (Event) = $\frac{\textbf{number of favorable outcomes}}{\textbf{number of possible outcomes}}$

   Theoretical Probability (3) = ____

2. What is the *experimental probability* of spinning a 3? To find the answer, the game must be played and the results of the spins recorded. Using the spinner below and the recording chart, play the game 20 times and record the results of each trial.



Directions for Making the Pointer

Straighten the large loop of a paperclip so that the end extends out from the rest of the paperclip's body.

Place the closed loop in the center of the circle.

Place the point of a pencil on the center of the circle so that is inside the paperclip's loop. Hold the pencil straight up in that position.

Spin the paperclip around the center of the circle (the pencil point) by flicking it with a finger.

Recording Chart

Number	Tally Marks	Total
1		
2		
3		

3. The formula below is used to organize the results of an experimental probability activity. Use the data from the spinner trials above to fill in the formula. Show the result as a ratio, decimal, and percent.

$$\textbf{Experimental Probability (Event)} = \frac{\textbf{number of favorable outcomes}}{\textbf{number of trials}}$$

Experimental Probability (3) = ______________________________

4. The more times an experimental probability activity is done, the closer its results will be to the predicted theoretical probability results. Gather results from the other groups that performed the spinner experiment, and record their results in the table below.

Group	Number of Times 3 Was Spun (Out of 20 Spins)
A (Your Group)	
B	
C	
D	
E	
TOTAL	

Experimental Probability (3) = ______________________________

5. Compare the results from the *theoretical* probability formula in step 1 with the results compiled from the *experimental* probability formula in step 4. Describe the difference between the two results.

____

____

____

____

Name: ANSWER KEY

The Spinner Game

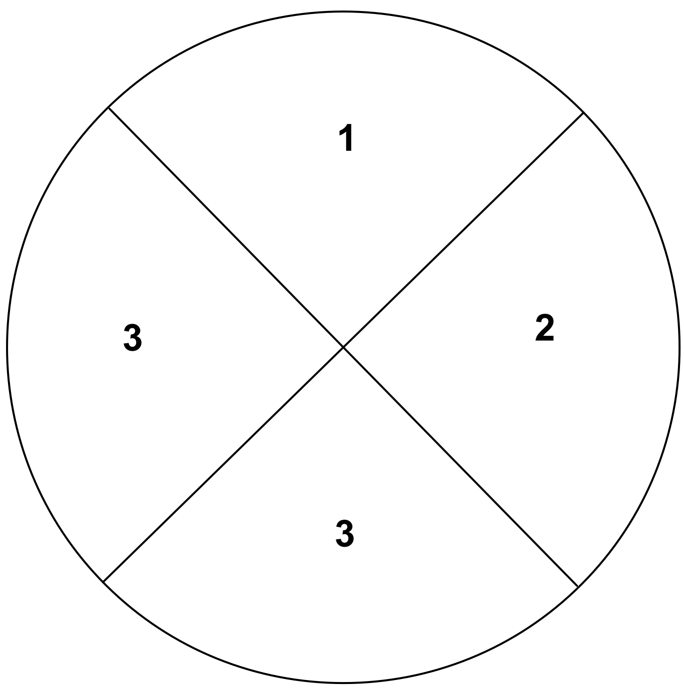
A spinner divided into four congruent parts is used in this game. (See the spinner below.) When the paperclip pointer lands on 3, the player earns a point.

1. What is the *theoretical probability* of spinning a 3? Use the formula below to calculate the theoretical probability of this event, and show the answer as a ratio, decimal (rounded to the nearest hundredth), and percent.

   Theoretical Probability (Event) = $\dfrac{\textbf{number of favorable outcomes}}{\textbf{number of possible outcomes}}$

   Theoretical Probability (3) = $\frac{2}{4} = \frac{1}{2}$ = 0.50 = 50%

2. What is the *experimental probability* of spinning a 3? To find the answer, the game must be played and the results of the spins recorded. Using the spinner below and the recording chart, play the game 20 times and record the results of each trial.



Directions for Making the Pointer

Straighten the large loop of a paperclip so that the end extends out from the rest of the paperclip's body.

Place the closed loop in the center of the circle.

Place the point of a pencil on the center of the circle so that is inside the paperclip's loop. Hold the pencil straight up in that position.

Spin the paperclip around the center of the circle (the pencil point) by flicking it with a finger.

Recording Chart

Number	Tally Marks	Total
1	(Answers will vary.)	
2		
3		

3. The formula below is used to organize the results of an experimental probability activity. Use the data from the spinner trials above to fill in the formula. Show the result as a ratio, decimal, and percent.

   $$\textbf{Experimental Probability (Event)} = \frac{\textbf{number of favorable outcomes}}{\textbf{number of trials}}$$

   Experimental Probability (3) = <u>(Answers will vary, but all of them should reflect the number of 3s in the tally sheet divided by 20.)</u>

4. The more times an experimental probability activity is done, the closer its results will be to the predicted theoretical probability results. Gather results from the other groups that performed the spinner experiment, and record their results in the table below.

Group	Number of Times 3 Was Spun (Out of 20 Spins)
A (Your Group)	
B	
C	
D	
E	
TOTAL	

   Experimental Probability (3) = <u>(Answers will vary, but all of them should reflect the number of 3s in the tally sheet divided by the total number of spins by all groups.)</u>

5. Compare the results from the *theoretical* probability formula in step 1 with the results compiled from the *experimental* probability formula in step 4. Describe the difference between the two results.

   <u>Answers will vary, but all of them should include the concept that theoretical probability is the mathematical chance of an event occurring, while experimental probability is the mathematical result of performing a series of trials of the event. Theoretical probability makes use of a formula to predict outcome, while experimental probability uses a formula to display the results of the experiment.</u>

   ____

   ____

   ____

Name: ________________________

Theoretical Probability and Experimental Probability

In the blank next to each situation listed below, place a *T* if the situation refers to theoretical probability or an *E* if it refers to experimental probability.

1. ____ Tomas has five cards that are numbered 1, 2, 3, 4, and 5. He turns them face down and says, “I have a 1 in 5, or 20%, chance of drawing a 5.”

2. ____ Tomas asks a friend draw to a card from his stack of five cards. The friend draws a 3. Tomas has him draw four more times and records the results.

3. ____ Three friends are playing a board game. In order to decide who will go first, they decide to roll a six-sided number cube to see who comes closest to rolling a 6, because they know that is a fair way to decide.

4. ____ Roseanne flips a coin four times in a row. She says, “If I flip it again, I have a 50% chance of it being heads.”

5. ____ Kim loves math. Whenever she has a free minute or two at the end of class, she rolls a six-sided number cube and records the results. She has recorded 500 results so far.

Name: ANSWER KEY

Theoretical Probability and Experimental Probability

In the blank next to each situation listed below, place a *T* if the situation refers to theoretical probability or an *E* if it refers to experimental probability.

1. _T_ Tomas has five cards that are numbered 1, 2, 3, 4, and 5. He turns them face down and says, "I have a 1 in 5, or 20%, chance of drawing a 5."

2. _E_ Tomas asks a friend draw to a card from his stack of five cards. The friend draws a 3. Tomas has him draw four more times and records the results.

3. _T_ Three friends are playing a board game. In order to decide who will go first, they decide to roll a six-sided number cube to see who comes closest to rolling a 6, because they know that is a fair way to decide.

4. _T_ Roseanne flips a coin four times in a row. She says, "If I flip it again, I have a 50% chance of it being heads."

5. _E_ Kim loves math. Whenever she has a free minute or two at the end of class, she rolls a six-sided number cube and records the results. She has recorded 500 results so far.

✱ SOL 7.15

Prerequisite SOL
6.20

Lesson Summary
Students identify and describe the number of possible arrangements of several items, using a tree diagram or the Fundamental (Basic) Counting Principle and a popular game used in decision making, Rock Paper Scissors. (50–60 minutes)

Materials
Copies of the attached worksheets
Index cards

Vocabulary
Fundamental (Basic) Counting Principle. A procedure to determine the number of possible arrangements of several objects; the product of the number of ways each object can be chosen individually.

Warm-up
Hand out copies of the “Combinations” worksheet, and allow students about five minutes to complete it. Then, discuss the students’ responses, displaying the answers on a transparency or the board. Ask students to state how many possible combinations (outcomes) Dante could choose.

Lesson
1. Explain how the game Rock Paper Scissors is played and ensure that all students understand it. (See Teacher Resource on the next page if you need a description of the game.) You may want to allow some time for students to practice playing it with a partner.
2. Distribute copies of the “Rock Paper Scissors” worksheet. Explain to the class that Rock Paper Scissors is a popular way to resolve a minor dispute between two parties or to decide which of two people might gain some favorable reward, like going first in a game. It is widely considered to be a fair way to make simple decisions because each of the two participants has an equal chance of winning.
3. Read the directions on the worksheet with the students. Answer any questions, and allow students time to complete the activity. Monitor student involvement as well as accuracy by moving from one student pair to another as they complete the tasks.
4. After students have completed the first page of the worksheet, pause and have a discussion about the results. Ask the students if the results surprised them. Have students share their observations including the comparison of the number of times each item was a winner. Ask the students why the number of ties was three while the number of wins for each item was two. Tell the students that being able to look at all possible outcomes of an event is important when it comes to analyzing the event.
5. Have students go to the next page on the worksheet and do the next task. Then, stop them to explain the Fundamental (Basic) Counting Principle. Show the students how what they just did for the task is the way this principle is used. (They should have multiplied 3 times 3 to arrive at 9 possible outcomes, which is the same number of outcomes obtained by using the tree diagram.) Explain that the advantage of the Fundamental (Basic) Counting Principle is that the number of possible outcomes of an event can be obtained very quickly. Ask students what they think is a disadvantage of the principle. They should say that with the principle, they can’t see or visualize the results like they can with a tree diagram.
6. Have students complete the worksheet. The last two tasks introduce a fourth item, Water, into the Rock Paper Scissors game. Students are asked to play the game, use the Fundamental (Basic) Counting Principle, and create a tree diagram.

7. Close the lesson by having the students share their results of the Rock Paper Scissors Water tasks. Ensure that students have correct answers, and clarify any misunderstandings.

Reflection

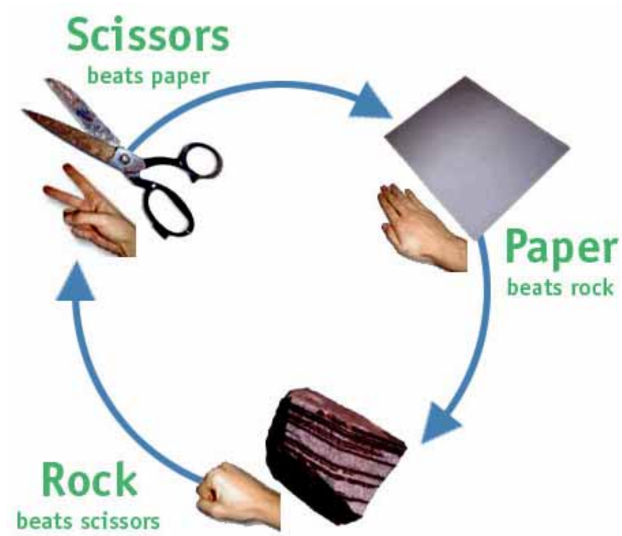
Give each student an index card, and ask students to write on it how many possible outcomes can be obtained by simultaneously rolling a six-sided number cube and tossing a coin (12). Students must show on the card how they obtained their answer. Encourage students to use the Fundamental (Basic) Counting Principle to find the answer.

Teacher Resource

Rock Paper Scissors is regarded as a way to make a decision between two alternatives when the decision to be made is of relatively low significance. This method of decision making is similar to flipping a coin or drawing straws. There are, however, certain mathematical principles associated with the game that make it fair.

The game is called a "hand game" in that the two players make hand signals or hand "throws." Traditionally, there are three hand signals:

- Closed fist = rock
- Open palm = paper
- Pointing finger and middle finger = scissors.



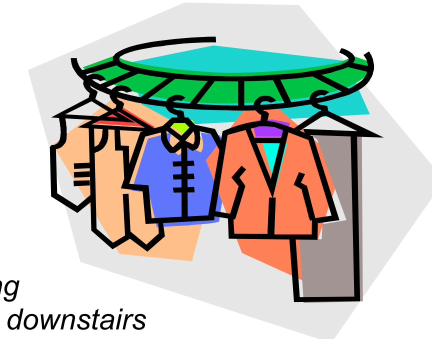
To play the game, each of the two players makes a fist with one hand. Together the players move their fists in an up and down motion four times while saying "Rock...Paper...Scissors...Go!" On "Go!," each player makes one of the hand-signal outcomes (rock, paper, or scissors) of their choice. The three possible outcomes have the following meanings:

- Rock breaks scissors.
- Paper wraps rock.
- Scissors cut paper.

Online Resources

The World RPS Society. http://www.worldrps.com/. This official Web site for the game explains that The World RPS Society is dedicated to the promotion of Rock Paper Scissors as a fun and safe way to resolve disputes.

Name: ________________________



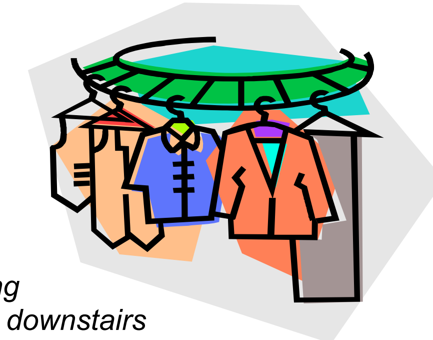
Combinations

Read the story problem.

> *Dante gets up late on Saturday morning. He enjoys being able to sleep in on a day off from school. When he goes downstairs to eat breakfast, his mother reminds him that she and Dante are going to visit Dante's grandmother today. After breakfast, Dante goes to his closet and pulls out a pair of tan pants and a pair of blue pants. He also gets three shirts — a red one, a green one, and a white one. He thinks, "I have several choices to make. I wonder how many different combinations of pants and shirts I could choose."*

Complete the tree diagram below by filling in the blanks with the possible choices Dante could make. Then, list the possible combinations (outcomes).

Tree Diagram (Sample Space)		**List of Possible Outcomes (Sample Space)**
	______shirt	____________________
______pants	______shirt	____________________
	______shirt	____________________
	______shirt	____________________
______pants	______shirt	____________________
	______shirt	____________________



Name: <u>ANSWER KEY</u>

Combinations

Read the story problem.

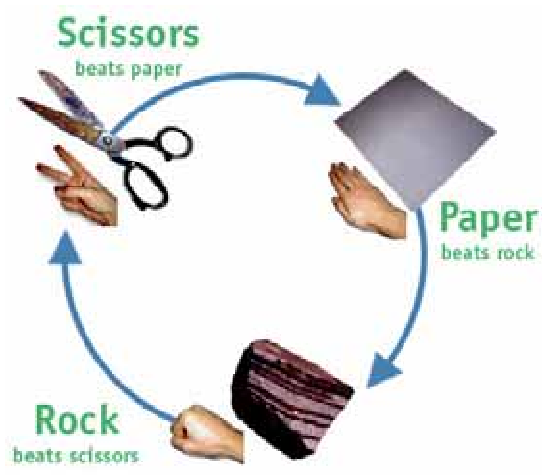
> *Dante gets up late on Saturday morning. He enjoys being able to sleep in on a day off from school. When he goes downstairs to eat breakfast, his mother reminds him that she and Dante are going to visit Dante's grandmother today. After breakfast, Dante goes to his closet and pulls out a pair of tan pants and a pair of blue pants. He also gets three shirts — a red one, a green one, and a white one. He thinks, "I have several choices to make. I wonder how many different combinations of pants and shirts I could choose."*

Complete the tree diagram below by filling in the blanks with the possible choices Dante could make. Then, list the possible combinations (outcomes).

Tree Diagram (Sample Space)		**List of Possible Outcomes (Sample Space)**
	red shirt	blue pants, red shirt
blue pants	green shirt	blue pants, green shirt
	white shirt	blue pants, white shirt
	red shirt	tan pants, red shirt
tan pants	green shirt	tan pants, green shirt
	white shirt	tan pants, white shirt

Name: ________________________

Rock Paper Scissors



Complete the tasks below. Record the answers in the spaces provided.

1. After you have played Rock Paper Scissors with a partner, do you think it's a *fair* game? That is, does each player have an equal chance of winning? To see whether this game is fair, create a tree diagram and a list of all possible outcomes. The first parts of the tree diagram and the list are done for you. Also, list the winner for each outcome.

Tree Diagram (Sample Space)		**List of Possible Outcomes (Sample Space)**	**Winner**
	Rock	Rock Rock	(Tie)
Rock	Paper	Rock Paper	Paper
	Scissors	Rock Scissors	Rock
	Rock	______________	________
______	Paper	______________	________
	Scissors	______________	________
	Rock	______________	________
______	Paper	______________	________
	Scissors	______________	________

2. Fill in the chart at right with the number of times each of the choices would win. Also, fill in the number of ties.

Number of Wins &Ties	
Rock	__
Paper	__
Scissors	__
Ties	__

3. How many possible outcomes are there? ___

4. What is the number of choices that Player A can make? ___

5. What is the number of choices that Player B can make? ___

6. Multiply these two numbers. What is the answer? ___

7. *This is the number of possible outcomes.* How does this number of possible outcomes compare with the number of possible outcomes in step 3 above? ____________________

Rock Paper Scissors Water

Rock Paper Scissors can be changed to Rock Paper Scissors Water. In this games, water can win twice because it can dissolve Paper and rust Scissors. Water can lose once because Rock can divide it. To signal Water, you cup your palm as if you were holding some water in it.

8. Play Rock Paper Scissors Water with a partner 10 times to see how it works. Record the results of your game in the chart at right.

Play	Player A Choice	Player B Choice	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

9. Use the Fundamental (Basic) Counting Principle to determine how many possible outcomes there are there in this game.

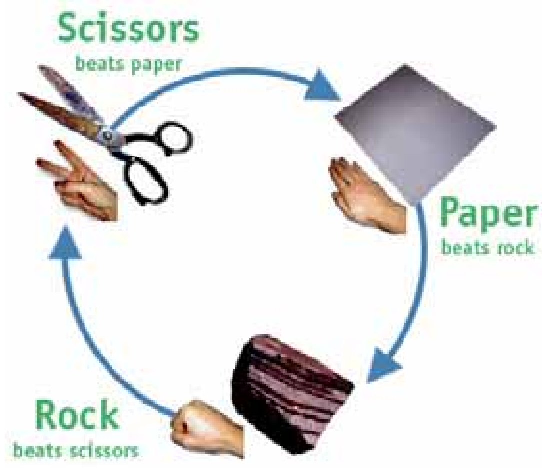
    ____

10. In the space below, complete the tree diagram and the list of possible outcomes. Also, list the winner for each outcome.

Tree Diagram (Sample Space)		List of Possible Outcomes (Sample Space)	Winner
	Rock	Rock Rock	(Tie)
Rock	Paper	Rock Paper	Paper
	Scissors	Rock Scissors	Rock
	Water	Rock Water	Rock
	_______	________________	_______
______	_______	________________	_______
	_______	________________	_______
	_______	________________	_______
	_______	________________	_______
______	_______	________________	_______
	_______	________________	_______
	_______	________________	_______
	_______	________________	_______
______	_______	________________	_______
	_______	________________	_______
	_______	________________	_______

Name: ANSWER KEY

Rock Paper Scissors



Complete the tasks below. Record the answers in the spaces provided.

1. After you have played Rock Paper Scissors with a partner, do you think it's a *fair* game? That is, does each player have an equal chance of winning? To see whether this game is fair, create a tree diagram and a list of all possible outcomes. The first parts of the tree diagram and the list are done for you. Also, list the winner for each outcome.

Tree Diagram (Sample Space)		List of Possible Outcomes (Sample Space)	Winner
	Rock	Rock Rock	(Tie)
Rock	Paper	Rock Paper	Paper
	Scissors	Rock Scissors	Rock
	Rock	Paper Rock	Paper
Paper	Paper	Paper Paper	(Tie)
	Scissors	Paper Scissors	Scissors
	Rock	Scissors Rock	Rock
Scissors	Paper	Scissors Paper	Scissors
	Scissors	Scissors Scissors	(Tie)

2. Fill in the chart at right with the number of times each of the choices would win. Also, fill in the number of ties.

Number of Wins & Ties	
Rock	2
Paper	2
Scissors	2
Ties	3

3. How many possible outcomes are there? 9

4. What is the number of choices that Player A can make? 3

5. What is the number of choices that Player B can make? 3

6. Multiply these two numbers. What is the answer? 9

7. *This is the number of possible outcomes.* How does this number of possible outcomes compare with the number of possible outcomes in step 3 above? They are the same.

Rock Paper Scissors Water

Rock Paper Scissors can be changed to Rock Paper Scissors Water. In this games, water can win twice because it can dissolve Paper and rust Scissors. Water can lose once because Rock can divide it. To signal Water, you cup your palm as if you were holding some water in it.

8. Play Rock Paper Scissors Water with a partner 10 times to see how it works. Record the results of your game in the chart at right. (Answers will vary.)

Play	Player A Choice	Player B Choice	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

9. Use the Fundamental (Basic) Counting Principle to determine how many possible outcomes there are there in this game.

   4 x 4 = 16

10. In the space below, complete the tree diagram and the list of possible outcomes. Also, list the winner for each outcome.

Tree Diagram (Sample Space)		**List of Possible Outcomes (Sample Space)**	**Winner**
	Rock	Rock Rock	(Tie)
Rock	Paper	Rock Paper	Paper
	Scissors	Rock Scissors	Rock
	Water	Rock Water	Rock
	Rock	Paper Rock	Paper
Paper	Paper	Paper Paper	(Tie)
	Scissors	Paper Scissors	Scissors
	Water	Paper Water	Water
	Rock	Scissors Rock	Rock
Scissors	Paper	Scissors Paper	Scissors
	Scissors	Scissors Scissors	(Tie)
	Water	Scissors Water	Water
	Rock	Water Rock	Rock
Water	Paper	Water Paper	Water
	Scissors	Water Scissors	Water
	Water	Water Water	(Tie)

✱ SOL 8.11

Prerequisite SOL

7.14, 7.15

Lesson Summary

Students analyze a simple game of chance involving the flipping of colored, plastic chips. They use previously learned principles of probability to predict the results of the game. (90 minutes)

Materials

Calculators
Copies of the attached worksheets
Sets of colored plastic chips (see “Chip-Tossing Game” worksheet)

Vocabulary

prediction. A reasonable guess as to what might happen in a certain situation, based on data gathered from a sample space.

theoretical probability. The predicted outcome of an event occurring, expressed as a ratio of the number of favorable outcomes divided by the number of possible outcomes; may be displayed as a ratio (fraction), decimal, or percent.

Warm-up

Distribute copies of the “True or False” worksheet, which will allow students to review basic probability concepts. Take time to discuss the answers to the situations. Be sure to clear up any confusion that students might have regarding the suggested solutions.

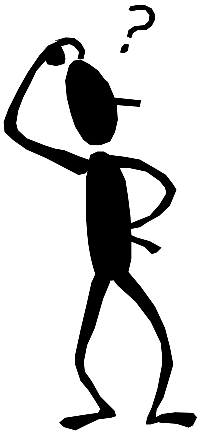
Lesson

1. Ask students how many of them have played games involving chance. Allow some time for them to share their experiences with such games. Try to guide students to say what makes such games fair or unfair. Games with spinners or games with number cubes, for example, take some knowledge of the concepts of probability in order to predict the chances of winning. The more a player understands his/her chances, the more opportunity there is for that player to win the game.
2. Explain to the class that they will read the rules for a particular game of chance and then predict what the outcomes will be. Distribute copies of the “Chip-Tossing Game” worksheet. Read the rules of the game aloud, and lead the students in making the Probability Observations. Demonstrate the game by playing it with the class. Give students the opportunity to practice playing the game while answering any questions they may have about the rules. Ask whether the game is fair, and have students explain why or why not.
3. Have students play the game. Write the results from each game played on a class results chart, using a format that allows for comparison of each player’s total score.
4. Conduct a discussion about all the results being displayed. This should include questions that allow students to see the advantages of taking the results and calculating the percentages of wins for Player 1 and Player 2. Remind students of the formula for calculating outcomes of probability, or winning percentage: i.e., the number of wins divided by the number of possible outcomes.
5. Distribute copies of the “New Game” worksheet, and read the directions with the students. Ask them to complete the first task, in which they find the winning percentage of the “Chip-Tossing Game,” using the whole-class results previously obtained.
6. After a brief discussion about the answers to this first task to ensure that all students performed it correctly, allow students time to complete the remainder of the tasks on the sheet. This should result in the completion of a tree diagram, the answering of a set of questions related to the tree diagram, and a modification of the rules of the “Chip-Tossing Game” in order to make it fair for both players.

7. As students finish the tasks on the "New Game" sheet, allot some time for sharing of results and discussing of student analyses that led to the creation of the "New Game."

Reflection

Distribute copies of the "Reflection" worksheet. Allow time for students to complete the task on it as an exit ticket to leave class.



Name: ________________________

TRUE or FALSE?

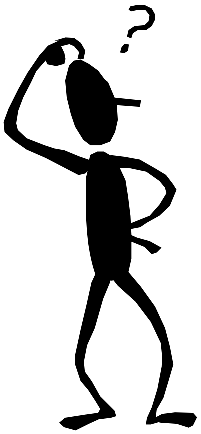
Read each of the following statements, and write TRUE or FALSE to indicate whether the statement is correct or not.

1. If a number cube with the numbers 1, 2, 3, 4, 5, and 6 is rolled, the chance of rolling a 1 is one out of six. ____________

2. If a number cube with the numbers 1, 2, 3, 4, 5, and 6 is rolled, the chance of rolling an even number is three out of six. ____________

3. If a pair of number cubes with the numbers 1, 2, 3, 4, 5, and 6 is rolled, the chance of rolling a sum of 3 is *greater* than the chance of rolling a sum of 7. ___________

4. If a coin is flipped three times and each time heads is the result, the probability of flipping tails on the next flip is 100%. ____________



Name: ANSWER KEY

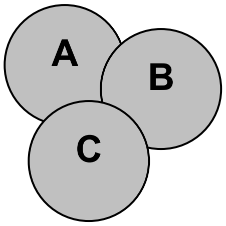
TRUE or FALSE?

Read each of the following statements, and write TRUE or FALSE to indicate whether the statement is correct or not.

1. If a number cube with the numbers 1, 2, 3, 4, 5, and 6 on it is rolled, the chance of rolling a 1 is one out of six. TRUE
2. If a number cube with the numbers 1, 2, 3, 4, 5, and 6 on it is rolled, the chance of rolling an even number is three out of six. TRUE (Some students may simplify "three out of six" to "one out of two," which is acceptable.)
3. If a pair of number cubes with the numbers 1, 2, 3, 4, 5, and 6 on them is rolled, the chance of rolling a sum of 3 is *greater* than the chance of rolling a sum of 7. FALSE
4. If a coin is flipped three times and each time heads is the result, the probability of flipping tails on the next flip is 100%. FALSE

Name: ________________________

Chip-Tossing Game



Materials
Three round plastic chips, labeled as follows:
- ***A*** on one side and ***B*** on the other side
- ***A*** on one side and ***C*** on the other side
- ***B*** on one side and ***C*** on the other side

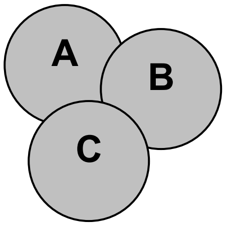
Probability Observations
1. What is the total number of sides? _____
2. How many A sides are there? _____
3. How many B sides are there? _____
4. How many C sides are there? _____

Directions
1. Play the game with a partner. First, decide who is Player 1 and who is Player 2.
2. Player 1 tosses all three chips into the air simultaneously and scores a point if any two chips match when they land. If none match (all three are different), Player 2 scores a point.
3. Place a tally mark in the score card below whenever a player scores a point.
4. The first player to get 20 points wins the game.

Player 1 (2 chips match = 1 pt)	Player 2 (all 3 chips different = 1 pt)

Name: <u>ANSWER KEY</u>

Chip-Tossing Game



Materials
Three round plastic chips, labeled as follows:
- ***A*** on one side and ***B*** on the other side
- ***A*** on one side and ***C*** on the other side
- ***B*** on one side and ***C*** on the other side

Probability Observations
1. What is the total number of sides? <u>6</u>
2. How many A sides are there? <u>2</u>
3. How many B sides are there? <u>2</u>
4. How many C sides are there? <u>2</u>

Directions
1. Play the game with a partner. First, decide who is Player 1 and who is Player 2.
2. Player 1 tosses all three chips into the air simultaneously and scores a point if any two chips match when they land. If none match (all three are different), Player 2 scores a point.
3. Place a tally mark in the score card below whenever a player scores a point.
4. The first player to get 20 points wins the game.

<u>(Sample Game)</u>
<u>(The first toss results in an outcome of ABA . Player 1 earns a point because two chips match. The second toss results in an outcome of ACB. Player 2 earns a point because all chips are different. Each player now has one tally mark representing the point that each earned. Players continue in the same way until one of them earns 20 points.)</u>

Player 1 (2 chips match = 1 pt)	Player 2 (all 3 chips different = 1 pt)
𝍸 𝍸 𝍸 𝍸	𝍸 𝍸 𝍷𝍷𝍷

Name: ________________________

New Game

Part 1
Complete the formulas below to find the winning percentage of the "Chip Tossing Game." Use the results displayed on the class results chart.

Player 1 winning percentage = $\dfrac{\text{number of wins}}{\text{number of possible outcomes}}$ = ___________ = _________%

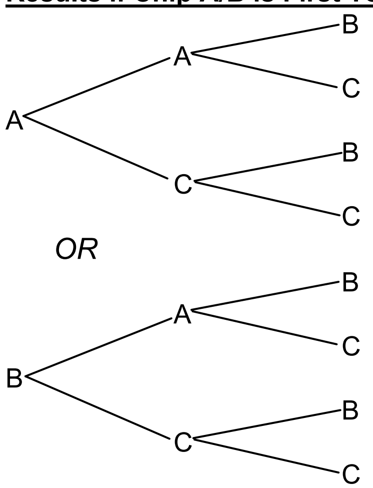
Player 2 winning percentage = $\dfrac{\text{number of wins}}{\text{number of possible outcomes}}$ = ___________ = _________%

Which player had the higher winning percentage? ______________

What is the difference between the percentages of the two players? ___________%

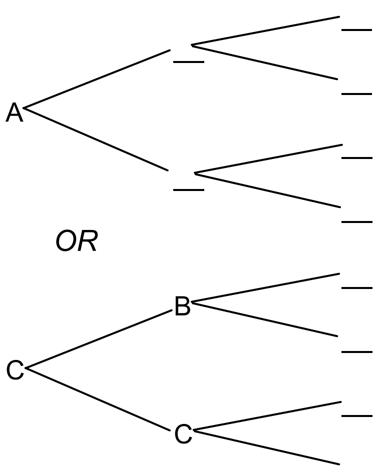
Part 2
Complete the tree diagram below showing the possible results of the "Chip Tossing Game."

Results if Chip A/B is First Toss



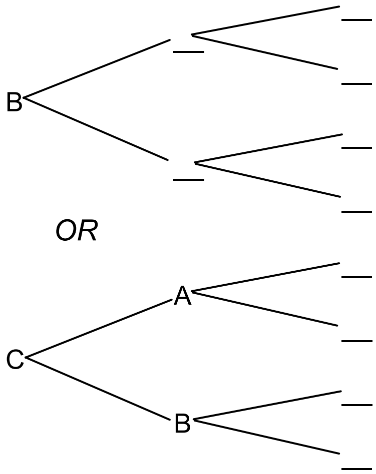
Possible Outcome	Winner
AAB	Player _____
AAC	Player _____
ACB	Player _____
ACC	Player _____
BAB	Player _____
BAC	Player _____
BCB	Player _____
BCC	Player _____

Results if Chip A/C is First Toss



Possible Outcome	Winner
ABA	Player _____
ABB	Player _____
ACA	Player _____
ACB	Player _____
CBA	Player _____
CBB	Player _____
CCA	Player _____
CCB	Player _____

Results if Chip B/C is First Toss



Possible Outcome	**Winner**
BAA	Player ____
BAC	Player ____
BBA	Player ____
BBC	Player ____
____	Player ____
CAC	Player ____
CBA	Player ____
____	Player ____

Complete the formula below to find the winning percentage of the “Chip Tossing Game.” Use the results displayed on the theoretical tree diagrams above in Part 2.

Player 1 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}}$ = __________ = ________%

Player 2 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}}$ = __________ = ________%

Which player should have the higher winning percentage? _____________

What is the difference between the percentages of the two players? __________%

Part 3

Change the rules of the “Chip Tossing Game” to allow it to be fair for both players. Write the rule change(s) here.

____

____

How do you know your rule change(s) will make the game fair for both players?

____

____

Complete the formulas below to find the winning percentages *for your New Game.* Create a tree diagram like the one in Part 2, using the outcomes section.

Player 1 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}}$ = __________ = ________%

Player 2 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}}$ = __________ = ________%

Use the Proof Sheet on the next page for any tree diagrams or score sheets that you need for your new game.

Name: <u>ANSWER KEY</u>

New Game

Part 1

Complete the formulas below to find the winning percentage of the "Chip Tossing Game." Use the results displayed on the class results chart. <u>(Answers will vary, but all results should give Player 1 a clear advantage.)</u>

Player 1 winning percentage = $\dfrac{\text{number of wins}}{\text{number of possible outcomes}}$ = __________ = ________%

Player 2 winning percentage = $\dfrac{\text{number of wins}}{\text{number of possible outcomes}}$ = __________ = ________%

Which player had the higher winning percentage? _____________

What is the difference between the percentages of the two players? __________%

Part 2

Complete the tree diagram below showing the possible results of the "Chip Tossing Game."

Results if Chip A/B is First Toss	Possible Outcome	Winner
A – A – B	AAB	Player 1
A – A – C	AAC	Player 1
A – C – B	ACB	Player 2
A – C – C	ACC	Player 1
OR		
B – A – B	BAB	Player 1
B – A – C	BAC	Player 2
B – C – B	BCB	Player 1
B – C – C	BCC	Player 1

Results if Chip A/C is First Toss	Possible Outcome	Winner
A – B – A	ABA	Player 1
A – B – B	ABB	Player 1
A – C – A	ACA	Player 1
A – C – B	ACB	Player 2
OR		
C – B – A	CBA	Player 2
C – B – B	CBB	Player 1
C – C – A	CCA	Player 1
C – C – B	CCB	Player 1

Results if Chip B/C is First Toss			**Possible Outcome**	**Winner**
B	A	A	BAA	Player 1
		C	BAC	Player 2
	B	A	BBA	Player 1
		C	BBC	Player 1
OR				
C	A	A	CCA	Player 1
		C	CAC	Player 1
	B	A	CBA	Player 2
		C	CBC	Player 1

Complete the formula below to find the winning percentage of the “Chip Tossing Game.” Use the results displayed on the theoretical tree diagrams above in Part 2.

Player 1 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}} = \frac{18}{24}$ = 75%

Player 2 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}} = \frac{6}{24}$ = 25%

Which player should have the higher winning percentage? Player 1

What is the difference between the percentages of the two players? 50%

Part 3
Change the rules of the “Chip Tossing Game” to allow it to be fair for both players. Write the rule change(s) here.

The best way to make the game fairer is to give 3 points to Player 2 when all of the letters are different. One point would still be given to Player 1 when two of the letters match.

How do you know your rule change(s) will make the game fair for both players?

(Answers will vary.)

Complete the formulas below to find the winning percentages *for your New Game.* Create a tree diagram like the one in Part 2, using the outcomes section. (Answers will vary.)

Player 1 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}}$ = ___________ = ________%

Player 2 winning percentage = $\frac{\text{number of wins}}{\text{number of possible outcomes}}$ = ___________ = ________%

Name: ________________________

Reflection

A six-sided number cube with the numbers 1, 2, 3, 4, 5, and 6 is used in a game. Each number is shown only once on the six sides of the cube.

Sam will win the game if he gets a number less than 3 the next time he rolls the number cube. What is the probability that Sam will win the game on his next roll? Circle your answer.

A $\frac{1}{6}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

In this box, *write* about how you solved the problem OR *draw a sketch or diagram* that explains the solution that you chose.

Name: <u>ANSWER KEY</u>

Reflection

A six-sided number cube with the numbers 1, 2, 3, 4, 5, and 6 is used in a game. Each number is shown only once on the six sides of the cube.

Sam will win the game if he gets a number less than 3 the next time he rolls the number cube. What is the probability that Sam will win the game on his next roll? Circle your answer.

A $\frac{1}{6}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

In this box, *write* about how you solved the problem OR *draw a sketch or diagram* that explains the solution that you chose.

<u>Student statements should explain that there are two possible winning rolls — a 1 or a 2 — and that there are six possible rolls. So 2 out of 6 times (or 1 out of 3 times, in simplified form), Sam will win.</u>

<u>A sketch could include a tree diagram that displays the six possible outcomes with two of them being winning rolls.</u>